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Article on the Non-homogeneous Sextic Equation with Five unknowns $3(x^3+y^3)(x-y)=103(z^2-w^2)T^4$

Anbuselvi.R[1] and Nandhini.R[2]

[1]Department of Mathematics, A.D.M.College for women (Autonomous), Nagapattinam, Tamil Nadu, India

[2]Department of Mathematics, Bharathidasan University Model College, Thiruthuraipoondi, Tamil Nadu, India

Abstract

The non-homogeneous sextic equation with five unknowns represented by $3(x^3+y^3)(x-y)=103(z^2-w^2)T^4$ is analysed for finding its patterns of non-zero distinct integral solutions. Using the linear transformations $x=u+v, y=u-v, z=3u+v, w=3u-v(u\neq v\neq 0)$ and applying the factorization method,six patterns and eight choices of non-zero distinct integral solutions are obtained. Along with the patterns, properties and some special numbers are presented.

Keywords: *Non-homogeneous equation, sextic equation, integral solutions, polygonal numbers, pyramidal numbers.*

1.Introduction

In [1,2] sextic equation with three unknowns are studied for its non-zero integral solutions. In [3-6], sextic equation with four unknowns are analysed. In particular, one may refer [7-9] for sextic equation with five unknowns. The Diophantine equation offers an unlimited field for research due to their more variety of problems[10-13]. In this article, the non-homogeneous sextic equation with five unknowns given by $3(x^3+y^3)(x-y)=103(z^2-w^2)T^4$ is considered analysed for finding its non-zero distinct integral solutions.Some of the interesting properties also obtained.

2.Notations Used

Polygonal number of rank n with size m

$$t_{m,n}=n[1+\frac{(n-1)(m-2)}{2}]$$

Pentagonal pyramidal number of rank n

$$P_n^5=\frac{n^2(n+1)}{2}$$

Four dimensional figurate number of rank n

$$FN_n^4=\frac{n^2(n^2-1)}{2}$$

Four dimensional figurate number of rank n whose generating polygon is a pentagon

$$F_{4,n,5}=\frac{3n^4+10n^3+9n^2+2n}{4!}$$

Nexus number of rank n

$$Nex_n=5n^4+10n^3+10n^2+5n+1$$

Centered square number

$$CS_n=n^2+(n-1)^2$$

Carol number, $Carol_n=(2^n-1)^2-2$

Mersenne Prime, $M_n=2^n-1$

3.Method of Analysis

The Diophantine equation representing the non-homogeneous sextic equation with five unknowns is represented by

$$3(x^3+y^3)(x-y)=103(z^2-w^2)T^4 \quad (1)$$

Introduction of the transformations

$$x=u+v, y=u-v, z=3u+v, w=3u-v, u\neq v\neq 0 \quad (2)$$

in (1) gives $u^2+3v^2=103T^4$ (3)

By approaching (3) in different ways and thus we obtain different patterns.Those patterns are given below:

3.Pattern 1

Let us take $T = a^2 + 3b^2$ (4)

Put $103 = (10 + i\sqrt{3})(10 - i\sqrt{3})$ (5)

Using (4)and(5) in (3) and using the method of factorization and equating positive factors,we get

$u + i\sqrt{3}v = (10 + i\sqrt{3})(a + i\sqrt{3}b)^4$ (6)

By equating real and imaginary parts,we get

$u = u(a,b) = 10a^4 - 12a^3b - 180a^2b^2 + 36ab^3 + 90b^4$ (7)

$v = v(a,b) = a^4 + 40a^3b - 18a^2b^2 - 120ab^3 + 9b^4$ (8)

Substituting (7)and(8) in (2) we get the following non-zero distinct integral solutions to (1)

$$x = x(a,b) = 11a^4 + 28a^3b - 198a^2b^2 - 84ab^3 + 99b^4$$

$$y = y(a,b) = 9a^4 - 52a^3b - 162a^2b^2 + 156ab^3 + 81b^4$$

$$z = z(a,b) = 31a^4 + 4a^3b - 558a^2b^2 - 12ab^3 + 279b^4$$

$$w = w(a,b) = 29a^4 - 76a^3b - 522a^2b^2 + 228ab^3 + 261b^4$$

$$T = T(a,b) = a^2 + 3b^2$$

Properties

1. $x(1,b) - 1188FN_b{}^4 + 198P_b{}^5 - 30CP_b{}^3 \equiv 11(mod13)$
2. $x(a,1) + y(a,1) - 240FN_a{}^4 + 16CP_a{}^9 + t_{562,a} + t_{122,a} \equiv 180(mod274)$
3. $y(a,1) + 13z(a,1) - 494FN_a{}^4 + t_{8008,a} + t_{6002,a} \equiv 3708(mod7002)$
4. $5T[a(a+1), a+2] - Nex_a - t_{14,a} - t_{10,a} \equiv 59(mod63)$
5. $z(1,b) - 3348FN_b{}^4 + 24P_b{}^5 + t_{270,b} + t_{268,b} \equiv 31(mod261)$
6. $w(1,b) - 3132FN_b{}^4 + 152CP_b{}^9 + t_{402,b} + t_{124,b} \equiv 29(mod259)$
7. $4T(a,a)$ is a perfect square.
8. $6T(a,a)$ is a nasty number.

4.Pattern 2

Take $103 = \frac{(20+i2\sqrt{3})(20-i2\sqrt{3})}{2^2}$ (9)

Using (4)and(5) in (3) and using the method of factorization and equating positive factors,we get

$u + i\sqrt{3}v = \frac{(20+i2\sqrt{3})(a+i\sqrt{3}b)^4}{2}$ (10)

Equating real and imaginary parts,we get $u = u(a,b) = \frac{1}{2}(20a^4 - 24a^3b - 360a^2b^2 + 72ab^3 + 180b^4)$

$v = v(a,b) = \frac{1}{2}(2a^4 + 80a^3b - 36a^2b^2 - 240ab^3 + 18b^4)$

For our convenient and to get integer solutions,we may take $a = 2A$, $b = 2B$

$u = u(a,b) = 2^3(20A^4 - 24A^3B - 360A^2B^2 + 72AB^3 + 180B^4)$ (11)

$v = v(a,b) = 2^3(2A^4 + 80A^3B - 36A^2B^2 - 240AB^3 + 18B^4)$ (12)

In view of (2),the non-zero distinct integral solutions to (1) are given by

$$x = x(A,B) = 2^3(22A^4 + 56A^3B - 396A^2B^2 - 168AB^3 + 198B^4)$$

$$y = y(A,B) = 2^3(18A^4 - 104A^3B - 324A^2B^2 + 312AB^3 + 162B^4)$$

$$z = z(A,B) = 2^3(62A^4 + 8A^3B - 1116A^2B^2 - 24AB^3 + 558B^4)$$

$$w = w(A,B) = 2^3(58A^4 - 152A^3B - 1044A^2B^2 + 456AB^3 + 522B^4)$$

$$T = T(A,B) = 2^2(A^2 + 3B^2)$$

Note

It is worth to mention here that 103 can also be represented in the following ways:

$$103 = \frac{(30+i3\sqrt{3})(30-i3\sqrt{3})}{3^2}$$

$$103 = \frac{(40+i4\sqrt{3})(40-i4\sqrt{3})}{4^2}$$

$$103 = \frac{(50+i5\sqrt{3})(50-i5\sqrt{3})}{5^2}$$ and so on

5.Pattern 3

Similarly $103 = \frac{(7+i11\sqrt{3})(7-i11\sqrt{3})}{2^2}$ (13)

Using (4) and(13) in (3),we get

$u + i\sqrt{3}v = \frac{(7+i11\sqrt{3})(a+i\sqrt{3}b)^4}{2}$

Equating real and imaginary parts,we get $u = u(a,b) = \frac{1}{2}(7a^4 - 132a^3b - 126a^2b^2 + 396ab^3 + 63b^4)$ $v = v(a,b) = \frac{1}{2}(11a^4 + 28a^3b - 198a^2b^2 - 84ab^3 + 99b^4)$

To get integer solutions, we may choose $a = 2A$, $b = 2B$

$u = u(a,b) = 2^3(7A^4 - 132A^3B - 126A^2B^2 + 396AB^3 + 63B^4)$ (14)

$v = v(a,b) = 2^3(11A^4 + 28A^3B - 198A^2B^2 - 84AB^3 + 99B^4)$ (15)

By putting (14) and (15) in (2) we obtain the following integral solutions to (1)

$$x = x(A,B) = 2^3(18A^4 - 104A^3B - 324A^2B^2 + 312AB^3 + 162B^4)$$

$$y = y(A,B) = 2^3(-4A^4 - 160A^3B + 72A^2B^2 + 480AB^3 - 36B^4)$$
$$z = z(A,B) = 2^3(32A^4 - 368A^3B - 576A^2B^2 + 1104AB^3 + 288B^4)$$
$$w = w(A,B) = 2^3(10A^4 - 424A^3B - 180A^2B^2 + 1272AB^3 + 90B^4)$$
$$T = T(A,B) = 2^2(A^2 + 3B^2)$$

6.Pattern 4

Mark $103 = \frac{(14+i22\sqrt{3})(14-i22\sqrt{3})}{4^2}$ (16)

Substituting (4),(16) in (3) and applying the same procedure followed in the previous patterns,we get

$$u + i\sqrt{3}v = \frac{(14+i22\sqrt{3})(a+i\sqrt{3}b)^4}{4}$$

Similarly we get

$$u = u(a,b) = \frac{1}{4}(14a^4 - 264a^3b - 252a^2b^2 + 792ab^3 + 126b^4)$$
$$v = v(a,b) = \frac{1}{4}(22a^4 + 56a^3b - 396a^2b^2 - 168ab^3 + 198b^4)$$

Chosen of $a = 4A$,$b = 4B$ leads to

$$u = u(a,b) = 4^3(14A^4 - 264A^3B - 252A^2B^2 + 792AB^3 + 126B^4) \quad (17)$$
$$v = v(a,b) = 4^3(22A^4 + 56A^3B - 396A^2B^2 - 168AB^3 + 198B^4) \quad (18)$$

Using (17) and (18) in (2), the values of x, y, z, w and T are given by

$$x = x(A,B) = 4^3(36A^4 - 208A^3B - 648A^2B^2 + 624AB^3 + 324B^4)$$
$$y = y(A,B) = 4^3(-8A^4 - 320A^3B + 144A^2B^2 + 960AB^3 - 72B^4)$$
$$z = z(A,B) = 4^3(64A^4 - 736A^3B - 1152A^2B^2 + 2208AB^3 + 576B^4)$$
$$w = w(A,B) = 4^3(20A^4 - 848A^3B - 360A^2B^2 + 2544AB^3 + 180B^4)$$
$$T = T(A,B) = 4^2(A^2 + 3B^2)$$

which are the non-zero distinct integral solutions of (1)

Note

Similarly one may write 103 in different ways:$103 = \frac{(21+i33\sqrt{3})(21-i33\sqrt{3})}{6^2}$

$$103 = \frac{(28+i44\sqrt{3})(28-i44\sqrt{3})}{8^2}$$
$$103 = \frac{(35+i55\sqrt{3})(35-i55\sqrt{3})}{10^2}$$ and so on

7.Pattern 5

One may write (3) as

$$u^2 + 3v^2 = 103T^4 * 1 \quad (19)$$

Also we may write

$$1 = \frac{(1+i4\sqrt{3})(1-i4\sqrt{3})}{7^2} \quad (20)$$

Using (4),(5) and (20) in (19),we get

$$u + i\sqrt{3}v = \frac{(10+i\sqrt{3})(1+i4\sqrt{3})(a+i\sqrt{3}b)^4}{4}$$
$$u = u(a,b) = \frac{1}{7}(-2a^4 - 492a^3b + 36a^2b^2 + 1476ab^3 - 18b^4)$$
$$v = v(a,b) = \frac{1}{7}(41a^4 - 8a^3b - 738a^2b^2 + 24ab^3 + 369b^4)$$

By choosing $a = 4A$, $b = 4B$ we get

$$u = u(a,b) = 7^3(-2A^4 - 492A^3B + 36A^2B^2 + 1476AB^3 - 18B^4) \quad (21)$$
$$v = v(a,b) = 7^3(41A^4 - 8A^3B - 738A^2B^2 + 24AB^3 + 369B^4) \quad (22)$$

Substituting (21),(22) in (2) we obtain non-zero integral solutions of (1)

$$x = x(A,B) = 7^3(39A^4 - 500A^3B - 702A^2B^2 + 1500AB^3 + 351B^4)$$
$$y = y(A,B) = 7^3(-43A^4 - 484A^3B + 774A^2B^2 + 1452AB^3 - 387B^4)$$
$$z = z(A,B) = 7^3(35A^4 - 1484A^3B - 630A^2B^2 + 4452AB^3 + 315B^4)$$
$$w = w(A,B) = 7^3(-47A^4 - 1468A^3B - 846A^2B^2 + 4404AB^3 - 423B^4)$$
$$T = T(A,B) = 7^2(A^2 + 3B^2)$$

8.Pattern 6

Eqn.(3) can be re-write as

$$u^2 - 100T^4 = 3[T^4 - v^2] \quad (23)$$

For this pattern, eight choices may arise which are given below:

8.1 Choice 1

Write (23) in the form of ratio as

$$\frac{u+10T^2}{T^2+v} = \frac{3(T^2-v)}{u-10T^2} = \frac{a}{b}, (b \neq 0)$$

which is equivalent to the following two equations

$$bu - av - (a - 10b)T^2 = 0$$
$$au + 3bv - (10a + 3b)T^2 = 0$$

By applying cross multiplication rule, we get

$$u = 10a^2 - 30b^2 + 6ab \quad (24)$$
$$v = -a^2 + 3b^2 + 20ab \quad (25)$$
$$T^2 = 3b^2 + a^2 \quad (26)$$

Eqn (26) is of the form $y^2 = Dx^2 + z^2$

$\therefore x = 2mn, y = m^2 + Dn^2, z = m^2 - Dn^2$

From this here,

$$a = m^2 - 3n^2$$
$$b = 2mn$$
$$T = m^2 + 3n^2$$

Substituting the values of a and b in (24),(25) and (26),we get

$u = 10m^4 + 12m^3n - 180m^2n^2 - 36mn^3 + 90n^4$

$v = -m^4 + 40m^3n + 18m^2n^2 - 120mn^3 - 9n^4$

Substituting the values of u and v in (2), we get the following distinct non-zero integral solutions to (1)

$$x = x(m,n) = 9m^4 + 52m^3n - 162m^2n^2 - 156mn^3 + 81n^4$$
$$y = y(m,n) = 11m^4 - 28m^3n - 198m^2n^2 + 84mn^3 + 99n^4$$
$$z = z(m,n) = 29m^4 + 76m^3n - 522m^2n^2 - 228mn^3 + 261n^4$$
$$w = w(m,n) = 31m^4 - 4m^3n - 558m^2n^2 + 12mn^3 + 279n^4$$
$$T = T(m,n) = m^2 + 3n^2$$

Properties

1.$z(m,1) + 19w(m,1) - 7416{FN_m}^4 + +t_{2002,m} + t_{1014,m} \equiv 5562(mod10504)$

2. $x(m,1) + 13w(m,1) - 4944{FN_m}^4 + +t_{10008,m} + t_{4004,m} \equiv 3708(mod7002)$

3. $y(m,1) - Nex_m - 24F_{4,m,5} - 36{FN_m}^4 + 96{P_m}^5 + t_{222,m} + t_{114,m} \equiv 11(mod87)$

4.$w(2^n,1) + 240 = 31M_{4n} - 4M_{3n} - 558M_{2n} + 12M_n$

5.$T(n,n-1) - CS_n - t_{36,n} + t_{32,n} \equiv 2(mod2)$

6. $T(2^n - 1,1) - 5 = Carol_n$

7.$-8[y(m,n)]$ is a biquadratic integer.

8. $T(m,m)$ is a perfect square.

8.2 Choice 2

Write (23) in the form of ratio as

$\frac{u+10T^2}{3(T^2+v)} = \frac{(T^2-v)}{u-10T^2} = \frac{a}{b}, (b \neq 0)$

which is equivalent to the following system of equations

$bu - 3av - (3a - 10b)T^2 = 0$

au+b$v - (10a + b)T^2 = 0$

By applying the method of cross multiplication rule, we get

$u = 30a^2 - 10b^2 + 6ab$ (27)

$v = -3a^2 + b^2 + 20ab$ (28)

$T^2 = b^2 + 3a^2$ (29)

Eqn (29) is satisfied by

$a = 2mn$

$b = m^2 - 3n^2$

$T = m^2 + 3n^2$

Substituting the values of a and b in (27),(28) and (29),we get

$u = -10m^4 + 12m^3n + 180m^2n^2 - 36mn^3 - 90n^4$

$v = m^4 + 40m^3n - 18m^2n^2 - 120mn^3 + 9n^4$

Substituting the values of u and v in (2), we get the following distinct non-zero integral solutions to (1)

$$x = x(m,n) = -9m^4 + 52m^3n + 162m^2n^2 - 156mn^3 - 81n^4$$
$$y = y(m,n) = -11m^4 - 28m^3n + 198m^2n^2 + 84mn^3 - 99n^4$$
$$z = z(m,n) = -29m^4 + 76m^3n + 522m^2n^2 - 228mn^3 - 261n^4$$
$$w = w(m,n) = -31m^4 - 4m^3n + 558m^2n^2 + 12m - 279n^4$$
$$T = T(m,n) = m^2 + 3n^2$$

8.3 Choice 3

Write (23) in the form of ratio as

$\frac{u+10T^2}{3(T^2-v)} = \frac{(T^2+v)}{u-10T^2} = \frac{a}{b}, (b \neq 0)$

which is equivalent to the following system of equations

$bu + 3av - (3a - 10b)T^2 = 0$

au-b$v - (10a + b)T^2 = 0$

By applying the method of cross multiplication rule, we get

$u = 30a^2 - 10b^2 + 6ab$ (30)

$v = 3a^2 - b^2 - 20ab$ (31)

$T^2 = b^2 + 3a^2$ (32)

Eqn (32) is satisfied by

$a = 2mn$

$b = m^2 - 3n^2$

$T = m^2 + 3n^2$

Substituting the values of a and b in (30),(31) and (32),we get

$u = -10m^4 + 12m^3n + 180m^2n^2 - 36mn^3 - 90n^4$

$v = -m^4 - 40m^3n + 18m^2n^2 + 120mn^3 - 9n^4$

Substituting the values of u and v in (2), we get the following distinct non-zero integral solutions to (1)

$$x = x(m,n) = -11m^4 - 28m^3n + 198m^2n^2 + 84mn^3 - 99n^4$$
$$y = y(m,n) = -9m^4 + 52m^3n + 162m^2n^2 - 156mn^3 - 81n^4$$
$$z = z(m,n) = -31m^4 - 4m^3n + 558m^2n^2 + 12mn^3 - 279n^4$$

$w = w(m,n) = -29m^4 + 76m^3n + 522m^2n^2 - 228mn^3 - 261n^4$

$T = T(m,n) = m^2 + 3n^2$

8.4 Choice 4

Write (23) in the form of ratio as

$\frac{u+10T^2}{T^2-v} = \frac{3(T^2+v)}{u-10T^2} = \frac{a}{b}, (b \neq 0)$

which is equivalent to the following system of equations

$bu + av + (10b - a)T^2 = 0$

$au - 3bv - (10a + 3b)T^2 = 0$

By applying the method of cross multiplication rule, we get

$u = 10a^2 - 30b^2 + 6ab$ (33)

$v = a^2 - 3b^2 - 20ab$ (34)

$T^2 = 3b^2 + a^2$ (35)

Eqn (35) is satisfied by

$a = m^2 - 3n^2$

$b = 2mn$

$T = m^2 + 3n^2$

Substituting the values of a and b in (33),(34) and (35),we get

$u = 10m^4 + 12m^3n - 180m^2n^2 - 36mn^3 + 90n^4$

$v = m^4 - 40m^3n - 18m^2n^2 + 120mn^3 + 9n^4$

Substituting the values of u and v in (2), we get the following distinct non-zero integral solutions to (1)

$x = x(m,n) = 11m^4 - 28m^3n - 198m^2n^2 + 84mn^3 + 99n^4$

$y = y(m,n) = 9m^4 + 52m^3n - 162m^2n^2 - 156mn^3 + 81n^4$

$z = z(m,n) = 31m^4 - 4m^3n - 558m^2n^2 + 12mn^3 + 279n^4$

$w = w(m,n) = 29m^4 + 76m^3n - 522m^2n^2 - 228mn^3 + 261n^4$

$T = T(m,n) = m^2 + 3n^2$

8.5 Choice 5

Write (23) in the form of ratio as

$\frac{u-10T^2}{T^2-v} = \frac{3(T^2+v)}{u+10T^2} = \frac{a}{b}, (b \neq 0)$

which is equivalent to the following system of equations

$bu + av - (a + 10b)T^2 = 0$

$au - 3bv + (10a - 3b)T^2 = 0$

By applying the method of cross multiplication rule, we get

$u = -10a^2 + 30b^2 - 6ab$ (36)

$v = a^2 - 3b^2 + 20ab$ (37)

$T^2 = 3b^2 + a^2$ (38)

Eqn (38) is satisfied by

$a = m^2 - 3n^2$

$b = 2mn$

$T = m^2 + 3n^2$

Substituting the values of a and b in (36),(37) and (38),we get

$u = -10m^4 + 12m^3n + 180m^2n^2 - 36mn^3 - 90n^4$

$v = m^4 + 40m^3n - 18m^2n^2 - 120mn^3 + 9n^4$

As the values of u and v are same as in choice 2,the non-zero integral values also same as in choice 2.

8.6 Choice 6

Write (23) in the form of ratio as

$\frac{u-10T^2}{3(T^2-v)} = \frac{T^2+v}{u+10T^2} = \frac{a}{b}, (b \neq 0)$

which is equivalent to the following system of equations

$bu + 3av - (3a + 10b)T^2 = 0$

$au - bv + (10a - b)T^2 = 0$

By applying the method of cross multiplication rule, we get

$u = -30a^2 + 10b^2 + 6ab$ (39)

$v = 3a^2 - b^2 + 20ab$ (40)

$T^2 = b^2 + 3a^2$ (41)

Eqn (41) is satisfied by

$a = 2mn$

$b = m^2 - 3n^2$

$T = m^2 + 3n^2$

Substituting the values of a and b in (39),(40) and (41),we get

$u = 10m^4 + 12m^3n - 180m^2n^2 - 36mn^3 + 90n^4$

$v = -m^4 + 40m^3n + 18m^2n^2 - 120mn^3 - 9n^4$

which is same as in choice 1.

Therefore the non-zero distinct integral values also same as in choice 1.

8.7 Choice 7

Eqn. (23) can be written in the form of ratio as

$\frac{u-10T^2}{T^2+v} = \frac{3(T^2-v)}{u+10T^2} = \frac{a}{b}, (b \neq 0)$

which is equivalent to the following set of equations

$bu - av - (a + 10b)T^2 = 0$

$au+3bv + (10a - 3b)T^2 = 0$

By applying the method of cross multiplication rule, we get

$u = -10a^2 + 30b^2 + 6ab$ (42)

$v = -a^2 + 3b^2 - 20ab$ (43)

$T^2 = 3b^2 + a^2$ (44)

Eqn (44) is satisfied by

$a = m^2 - 3n^2$

$b = 2mn$
$T = m^2 + 3n^2$
Substituting the values of a and b in (42),(43) and (44),we get

$$u = -10m^4 + 12m^3 n + 180m^2n^2 - 36mn^3 - 90n^4$$

$$v = -m^4 - 40m^3n + 18m^2n^2 + 120mn^3 - 9n^4$$

which is same as in choice 3.
Therefore the non-zero distinct integral values also same as in choice 3.

8.8 Choice 8

Write (23) in the form of ratio as

$$\frac{u-10T^2}{3(T^2+v)} = \frac{T^2-v}{u+10T^2} = \frac{a}{b}, (b \neq 0)$$

which is equivalent to the following double equations

$$bu + av + (10b - a)T^2 = 0$$

$$au - 3bv - (10a + 3b)T^2 = 0$$

By applying the method of cross multiplication rule, we get

$$u = 10a^2 - 30b^2 + 6ab \quad (45)$$

$$v = a^2 - 3b^2 - 20ab \quad (46)$$

$$T^2 = 3b^2 + a^2 \quad (47)$$

Eqn (47) is satisfied by
$a = m^2 - 3n^2$
$b = 2mn$
$T = m^2 + 3n^2$
Substituting the values of a and b in (45),(46) and (47),we get

$$u = 10m^4 + 12m^3n - 180m^2n^2 - 36mn^3 + 90n^4$$

$$v = m^4 - 40m^3n - 18m^2n^2 + 120mn^3 + 9n^4$$

which is same as choice 4.
Therefore the non-zero distinct integral values also same as choice 4.

9.Conclusions

It is worth to note here that one may use some other transformations to obtain different patterns.To conclude one may try some other forms of homogeneous or non-homogeneous sextic equation with more than five variables and search for their integral solutions and properties.

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